Global finite-time stabilization for a class of switched nonhomogeneous positive systems via impulsive control

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***Abstract*—This paper studies the global finite-time stabilization problem for a type of switched nonhomogeneous positive nonlinear systems using impulsive control. Based on positive system theory and impulsive control technology, we introduce suitable impulsive inputs at switching points and design a class of stabilizing impulses coupled with maximum dwell-time (MDT) conditions, under which the considered system is globally finite-time stable. Meanwhile, the upper bound of settling-time function can be derived under MDT conditions, which uncovers the relationship among impulse intensity, initial value and control frequency. Finally, an example is presented to verify our result.**

***Index Terms*—Switched systems, positive systems, impulsive control, finite-time stable.**

I. Introduction

In numerous fields such as chemistry, ecology and communications, there exists a class of systems whose states remain nonnegative at all times, which are called positive systems [1]. With the rapid development of positive system theory, some basic problems have been widely discussed, especially stability analysis and stabilization of positive linear systems [2]–[5]. However, many real-world systems are relatively complex and often exhibit some unique nonlinear properties. Therefore, scholars have recently started to focus on various kinds of homogeneous positive nonlinear systems (HPNs) and develop new powerful tools [6]–[9].

In recent years, a class of continuous-time positive nonlinear systems defined by cooperative homogeneous vector fields has been proposed and received much attention, since such systems can be regarded as natural extensions of positive linear systems [10]–[12]. Later, some effective methods are proposed to solve some fundamental problems for such systems with any degrees $a > 0$. For example, the authors in [8] consider three cases for homogeneous positive systems according to the magnitude of degrees and give the corresponding stability criteria. This result suggests that HPNs with different degrees may exhibit diverse stability properties. In particular, when the degree is equal to one, HPNs may be globally exponentially stable, while when the degree is less than one, HPNs may converge to zero in finite time, i.e., it is finite-time stable (FTS). In [13], the stability problem of impulsive HPN systems of degree one with time delay is handled by utilizing max-separable Lyapunov functions (MSLFs) and Razumikhin technique. Subsequently, scholars have further focused on switched positive nonlinear systems (SPNSs) consisting of multiple HPN modes (subsystems) and a switching mechanism (see, e.g., [14]–[18]). To date, various constrained switching signals have been developed to study the stability of SHPNs. For instance, several stability conditions for SPNSs with average dwell-time (ADT) switching are derived without the aid of any Lyapunov functions in [14], where vector fields of all modes are homogeneous of degree one. Recently, based on piecewise MSLFs techniques, the authors in [19] present an explicit condition for global polynomial stability of SPNSs with degrees greater than one by constructing a class of logarithm contraction ADT switching.

However, it is important to acknowledge that most results on SPNSs assume that each mode is comprised solely of homogeneous vector fields with equal degrees. In practical scenarios, the aforementioned assumption can be overly stringent and often contradicts reality. Therefore, as opposed to previous research [14]–[19], our work considers a family of nonhomogeneous SPNSs with hybrid degrees, where each mode is formed by two homogeneous nonlinear parts: one of terms has a degree greater than one and the other equal to one. Specifically, when a mode contains unstable homogeneous term with degree equal to one, it may no longer inherit the globally finite-time stability inherent in stable homogeneous term of degree less than one, and may even diverge. How developing the stability and stabilization of such

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nonhomogeneous SPNSs is a challenging problem worthy of further study. Regrettably, this area of research still remains significantly underexplored. Additionally, impulsive control, as a discontinuous control technique, applies input to the system only at discrete moments and boasts low cost and strong robustness advantages [20]–[22]. Notably, stabilizing impulses can be regarded as stable switching under certain conditions, and have found wide application in the control of various switched systems [23]–[25]. This also provides a feasible solution for the finite-time stabilization of nonhomogeneous SPNSs.

Motivated by above discussions, we shall study the global finite-time stabilization problem for nonhomogeneous SNPSs with stabilizing impulses from the perspective of impulsive control in this paper. Based on multiple MSLFs and impulsive system theory, we design a class of impulsive-switching signals with MDT conditions and suitable impulsive controllers under which states can reach the origin in finite time, where impulse jumps occur at switching instants. Meanwhile, we also offer an estimate of settling time, which depends on MDT constant and initial value.

Notations: Let $\mathbb{R}^d$ and $\mathbb{R}^{d\times r}$ denote d-dimensional vectors space and matrix set, respectively. $\mathbb{N}_+ = \{0,1,2,\cdots\}$, $\mathbb{N} = \mathbb{N}_+\backslash\{0\}$ and $\Pi \triangleq \{1,\cdots,n\}$. Define $\upsilon \preceq \omega$ ($\upsilon \prec \omega$) for given $\upsilon = (\upsilon_j) \in \mathbb{R}^n$, $\omega = (\omega_j) \in \mathbb{R}^n$ if $\upsilon_j \le \omega_j$ ($\upsilon_j < \omega_j$), for all $j \in \Pi$. Moreover, for two vectors $\upsilon, \theta \succ 0$, we denote $\|\upsilon\|_\infty^\theta = \max_{j\in\Pi}\{\frac{\upsilon_j}{\theta_j}\}$. A matrix $D = [d_{ij}]_{n\times n} \in \mathbf{M}$, if $d_{ij} \ge 0, i \ne j, i,j \in \Pi$. Let $\mathbb{R}_+^n \triangleq \{\upsilon \in \mathbb{R}^n : \upsilon_i \ge 0, i \in \Pi\}$, and $\mathbb{R}_+^{n\times n} \triangleq \{D = [d_{ij}] \in \mathbb{R}^{n\times n} : d_{ij} \ge 0, i,j \in \Pi\}$.

II. Problem Formulation

Consider a class of nonhomogeneous SPNSs with H modes:

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t)), \quad t \ge 0 \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}_+^n$ denotes state of system (1), and for each $q \in \Omega = \{1,\cdots,H\}, H > 1$, vector fields $f_q(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ and $g_q(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ are continuous and $f_q(0) = g_q(0) = 0$. The map $\sigma(t) : [0,\infty) \to \Omega$ is a switching rule and Ω represents the index set of submodes. For a given $\sigma(t)$, the corresponding switching sequence $\{s_k\}_{k\in\mathbb{N}_+}$ meets $0 < s_1 < s_2 < \cdots < s_k$ as $k \to +\infty$, i.e., without Zeno behavior in system (1), and such type of switching signals is defined by set S for later use.

Definition 1. *A function $h(x) : \mathbb{R}^n \to \mathbb{R}^n$ is called homogeneous of degree $a > 0$, if for $\forall x \in \mathbb{R}^n$ and $a > 0$, it has $h(rx) = r^a h(x)$. Specifically, if $a = 1$, i.e., $h(rx) = rh(x)$, $h(x)$ is said to be homogeneous.*

Definition 2. *A function $h(x) : \mathbb{R}^n \to \mathbb{R}^n$, which continuously differentiable on $\mathbb{R}^n\backslash\{0\}$, is called cooperative if the Jacobian matrix $\frac{\partial h}{\partial x}(\tau) \in \mathbf{M}$ for $\forall \tau \in \mathbb{R}_+^n\backslash\{0\}$.*

Lemma 1. *For given cooperative function $h(x)$ and $\varphi, \psi \in \mathbb{R}^n\backslash\{0\}$, if $\varphi \preceq \psi$ with $\varphi_l = \psi_l, l \in \Pi$, then $h_l(\varphi) \le h_l(\psi)$.*

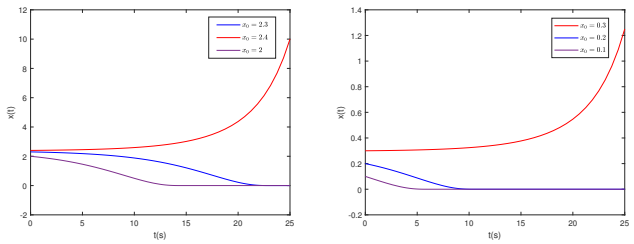
Assumption 1. *The system (1) satisfies the following assumptions:*

- (H_1) *For each mode $q \in \Omega$, $f_q(x)$ is homogeneous of $a \in (0,1)$ and $g_q(x)$ is homogeneous;*
- (H_2) *For each mode $q \in \Omega$, $f_q(x)$ and $g_q(x)$ are cooperative;*
- (H_3) *For each mode $q \in \Omega$, there exists a vector $\theta_q \succ 0$ such that $f_q(\theta_q) \prec 0 \prec g_q(\theta_q)$.*

Remark 1. *Assumption 1 implies that system (1) is positive, i.e., $x(t) \succeq 0$, see reference [26]. It is noted that, unlike homogeneous positive nonlinear systems studied in most works, each mode of system (1) considered is made up of two nonlinear terms with different degrees of homogeneity. In this case, system (1) is essentially inhomogeneous, which undoubtedly complicates stability analysis. Specifically, if $a = 1$, then system (1) is a homogeneous SPNS. Therefore, according to the result in [13], if for mode $p \in \Omega$, there exists a positive vector θ_p such that*

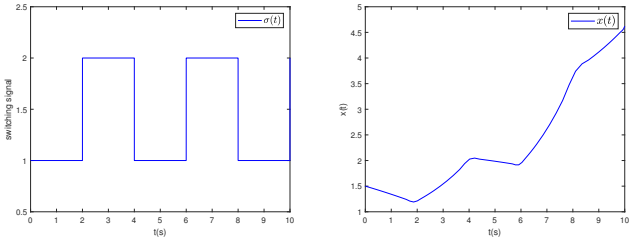
$$f_p(\theta_p) + g_p(\theta_p) \prec 0 \tag{2}$$

then mode p of system (1) is globally exponentially stable. Reference [8] shows that if $g_p(x) = 0, p \in \Omega$ and condition (2) holds, then mode p of system (1) is globally FTS. However, these conclusions are invalid for nonhomogeneous system (1) with different degrees under Assumption 1. That is system (1) may diverge even if vector fields f_p and g_p meet (2), which can be verified in Example 1.



(a) mode 1 (b) mode 2

Fig. 1: Trajectories of system (3) with different x_0.



(a) switching signal $\sigma(t)$ (b) curve of $x(t)$

Fig. 2: Trajectories of system (3) with $x_0 = 1.5$.

Example 1. *Consider a one-dimensional SPNS:*

$$\begin{cases} \textit{mode 1: } \dot{x}(t) = -0.8x^{\frac{2}{3}}(t) + 0.6x(t) \\ \textit{mode 2: } \dot{x}(t) = -0.4x^{\frac{2}{3}}(t) + 0.6x(t) \end{cases} \tag{3}$$

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where mode 1 and 2 are nonhomogeneous and satisfy Assumption 1. In fact, inequality (2) holds for system (3), for example we can take $\theta_1 = 1$ and $\theta_2 = 0.1$. However, Fig. 1 shows that two modes are locally FTS. Obviously, this is due to the existence of unstable homogeneous terms in system (3), which destroys the finite-time stability of homogeneous term of degree $a \in (0,1)$. Specially, mode 1 is FTS and mode 2 is opposite for $x_0 = 1.5$. Fig. 2(b) illustrates impulse-free system (3) with unsuitable switching $\sigma(t)$ (see Fig. 2(a)) is not FTS.

Therefore, to ensure the global finite-time stability of such systems, we introduce a class of mode-dependent stabilizing impulses $I_p(x)$ from the perspective of impulsive control. The SPNS (1) is rewritten as

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t)), & t \neq s_k \\ x(t) = I_{\sigma(t^-)}(x(t^-)), & t = s_k, k \in \mathbb{N}_+ \end{cases} \tag{4}$$

where s_k represents the impulsive-switching instant, that is, switching and impulses are synchronized. For each $q \in \Omega$, the impulsive function $I_q(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ is continuous and $I_q(x) = 0$ iff $x = 0$. Throughout this article, we assume that solution $x(t)$ uniquely exists in forward time under proper conditions, and $x(t)$ is right continuous and its left limit exists.

The objective of this work is to design suitable impulsive-switching laws and impulsive control inputs $I_q(x)$ simultaneously, and establish a relationship among on $I_q(x)$ and switching/impulse frequency, to ensure finite-time stability properties of system (4).

Definition 3. *For given impulsive-switching signal $\sigma(t) \in S$ and $x_0 \in \mathbb{R}^n_+$, system (4) is said to be globally finite-time stable (FTS), if*

- (a) *system (4) is Lyapunov stable (LS);*
- (b) *for any given $x_0 \in \mathbb{R}^n_+ \backslash \{0\}$, there exists a settling-time $0 < T(x_0, \sigma) < \infty$, such that $x(t) \equiv 0$ for $\forall t \geq T(x_0, \sigma)$.*

III. Main Results

In this section, we shall present some sufficient conditions for global finite-time stabilization of system (4) with stabilizing impulses. To do this, we first design a class of maximum dwell-time (MDT) impulsive-switching signals $S_{\tau_m} \in S$ satisfying $s_{k+1} - s_k \leq \tau_m$ for all $k \in \mathbb{N}$, where $\tau_m > 0$ is a finite scalar and called MDT constant.

To study the global finite-time stabilization of system (4), the following Assumption is given.

Assumption 2. *For $\forall q \in \Omega$ and $x \in \mathbb{R}^n_+$, $I_q(x) \succeq 0$ and there exist matrices $D_q \in \mathbb{R}^{n\times n}_+$ satisfying $I_q(x) \preceq D_q x$.*

The system (4) is clearly positive under Assumption 1 and 2. With the help of positive system theory and impulsive control strategies, we have the following result.

Theorem 1. *Consider system (4) with stabilizing impulses, and suppose that there exist a set of n-dimensional vectors $\theta_p \succ 0$ and two scalar $\tau_m > 0$, $d \in (0,1)$ such that Assumption 1-2 and*

$$D_q \theta_q \preceq d\theta_p \tag{5}$$

hold for all $p \neq q, p, q \in \Omega$. Then for all $x_0 \succeq 0$, system (4) is FTS under the class S_{τ_m} with

$$\tau_m \leq -\frac{\ln d}{2\beta} \tag{6}$$

Moreover, the settling-time $T(x_0, \sigma)$ is estimated by

$$T(x_0, \sigma) \leq \frac{1}{\beta(1-a)} + N_d \tau_m \tag{7}$$

and

$$\begin{cases} N_d \triangleq \inf\{l \in \mathbb{N} : d^{(1-a)l} - \triangle^2 \leq 0\} \\ \triangle = \dfrac{\lambda - 2\beta\tilde{\omega}_0 + \sqrt{(2\beta\tilde{\omega}_0 - \lambda)^2 + 4\lambda^2}}{2\lambda} \end{cases} \tag{8}$$

where $\tilde{\omega}_0 = (\|x_0\|_\infty^{\theta_{\sigma(0)}})^{1-a}$ and

$$\lambda = \min_{q\in\Omega, i\in\Pi} \left\{ -\frac{f_{qi}(\theta_q)}{\theta_{qi}} \right\}, \quad \beta = \max_{q\in\Omega, i\in\Pi} \left\{ \frac{g_{qi}(\theta_q)}{\theta_{qi}} \right\}$$

Proof. First, for given $x_0 \succeq 0$ and $\sigma(t)$, we can have $x(t) \succeq 0$ under Assumption 1 and 2. Thus, we can choose the following MSLFs

$$V_q(x(t)) = \max_{i\in\Pi} \frac{x_i(t)}{\theta_{qi}} \geq 0 \tag{9}$$

where $\theta_q \succ 0, q \in \Omega$ fulfills Assumption 1. Let $\omega(t) := V_{\sigma(t)}(x(t))$ and $\omega_0 = \omega(0)$. Moreover, by the definition of $\omega(t)$, there exists a index $i \in \Pi$ such that

$$x(t) \preceq \omega(t)\theta_{\sigma(t)}, \quad x_i(t) = \omega(t)\theta_{\sigma(t)i} \tag{10}$$

In what follows, we shall demonstrate the finite-time stability property of system (4) by discussing the following two situations.

Case 1: when $\omega_0^{1-a} \leq \frac{\lambda}{2\beta}$, under designed impulsive-switching laws $\sigma(t) \in S_{\tau_m}$ satisfying MDT condition (6) and impulsive control, the following conclusion

$$x(t) \equiv 0, \quad \text{for } t \geq \frac{1}{\beta(1-a)} > 0 \tag{11}$$

is true. In fact, we can derive from Assumption 1 and (5) that

$$\omega^{1-a}(t) \leq \frac{\lambda}{2\beta}, \quad \forall t \geq 0 \tag{12}$$

If inequality (12) is not valid, then there must exist a point t^* at which $\omega^{1-a}(t^*) > \lambda/(2\beta)$. Furthermore, we can conclude that

$$\begin{aligned} D^+ V_q(t) &= \frac{\dot{x}_l(t)}{\theta_{ql}} = \frac{f_{ql}(x(t)) + g_{ql}(x(t))}{\theta_{ql}} \\ &\leq \frac{f_{ql}(\theta_q)\omega^a(t) + g_{ql}(\theta_q)\omega(t)}{\theta_{ql}} \\ &\leq -\lambda V_q^a(t) + \beta V_q(t), \quad \forall t \in [s_k, s_{k+1}) \end{aligned} \tag{13}$$

where $\sigma(s_k) = q$ and $\sigma(s_k^-) = p$. The penultimate inequality of (13) is derived from Assumption 1 and inequality (10).

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For another, for impulsive-switching point s_k, one has

$$\begin{aligned} V_q(x(s_k)) &= \max_{i\in\Pi} \frac{I_{pi}(x(s_k^-))}{\theta_{qi}} \\ &\le \max_{i\in\Pi} \frac{(D_p x(s_k^-))_i}{\theta_{qi}} \\ &\le \max_{i\in\Pi} \frac{(D_p\theta_p)_i}{\theta_{qi}} \cdot V_p(x(s_k^-)) \\ &\le dV_p(x(s_k^-)) \end{aligned} \tag{14}$$

The last inequality of (14) is obtained from condition (5). In this case, we denote

$$\hat{t} := \inf\{t \ge 0 \mid \omega^{1-a}(t) > \lambda/(2\beta)\}$$

Obviously, $\hat{t} \notin \{s_k\}_{k\in\mathbb{N}_+}$ since $d < 1$. Thus, one can get

$$\omega^{1-a}(\hat{t}) = \lambda/(2\beta), \quad \omega^{1-a}(t) \le \lambda/(2\beta), \quad \forall t \in [0, \hat{t}]$$

and $D^+\omega(\hat{t}) \ge 0$. However, it follows from (13) that

$$\begin{aligned} D^+\omega(\hat{t}) &\le -\lambda\omega^a(\hat{t}) + \beta\omega^{1-a}(\hat{t}) \cdot \omega^a(\hat{t}) \\ &\le -\lambda\left(1 - \frac{1}{2}\right)\omega^a(\hat{t}) \\ &= -\frac{\lambda}{2}\omega^a(\hat{t}) \end{aligned} \tag{15}$$

which implies $D^+\omega(\hat{t}) < 0$, that is, we get a contradiction. Hence, we conclude that inference (12) is true, and further

$$\begin{aligned} D^+\omega(t) &\le -\lambda\omega^a(t) + \beta\omega^{1-a}(t) \cdot \omega^a(t) \\ &\le -\frac{\lambda}{2}\omega^a(t), \quad t \ge 0 \end{aligned} \tag{16}$$

i.e., system (4) is LS. We then get from (16) that

$$\omega^{1-a}(t) \le \omega_0^{1-a} - \frac{\lambda}{2}(1-a)t, \quad \forall t \in [0, s_1) \tag{17}$$

When $t \in [s_1, s_2)$, it follows from (14) and (17) that

$$\begin{aligned} \omega^{1-a}(t) &\le \omega^{1-a}(s_1) - \frac{\lambda}{2}(1-a)(t-s_1) \\ &\le d^{1-a}\left(\omega_0^{1-a} - \frac{\lambda}{2}(1-a)s_1\right) - \frac{\lambda}{2}(1-a)(t-s_1) \\ &\le \omega_0^{1-a} - \frac{\lambda}{2}(1-a)t \end{aligned}$$

By repeating the above, we can derive that

$$\omega^{1-a}(t) \le \omega_0^{1-a} - \frac{\lambda}{2}(1-a)t, \quad \forall t \ge 0 \tag{18}$$

which implies that $\omega(t)$ reaches zero before time

$$T(x_0, \sigma) \le \frac{2\omega_0^{1-a}}{\lambda(1-a)} \le \frac{1}{\beta(1-a)} \tag{19}$$

That is to say, conclusion (11) holds and system (4) with impulsive-switching signal $\sigma(t) \in \boldsymbol{S}_{\tau_m}$ satisfying MDT condition (6) is FTS for $\omega_0^{1-a} \le \frac{\lambda}{2\beta}$, where settling-time $T(x_0, \sigma)$ satisfies (7).

Case 2: when $\omega_0^{1-a} > \frac{\lambda}{2\beta}$, multiply both sides of (13) by $\omega^{-a}(t)$, and applying inequality (13) on $[0, s_1)$ gives that

$$\begin{aligned} \omega^{1-a}(t) &\le e^{\beta(1-a)t}\omega_0^{1-a} - e^{\beta(1-a)t}\frac{\lambda}{\beta} + \frac{\lambda}{\beta} \\ &\le \left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right)e^{\beta(1-a)t} + \frac{\lambda}{\beta} - e^{\beta(1-a)t}\frac{\lambda}{2\beta} \\ &\le \left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right)e^{\beta(1-a)t} + \frac{\lambda}{2\beta}, \ \forall t \in [0, s_1) \end{aligned} \tag{20}$$

which together with (14) yields that $\forall t \in [s_1, s_2)$

$$\begin{aligned} \omega^{1-a}(t) \le& e^{\beta(1-a)(t-s_1)}\left(\omega^{1-a}(s_1) - \frac{\lambda}{\beta}\right) + \frac{\lambda}{\beta} \\ \le& e^{\beta(1-a)(t-s_1)}\left(d^{1-a}e^{\beta(1-a)s_1}\left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right)\right) \\ &- e^{\beta(1-a)(t-s_1)}\frac{\lambda}{\beta} + d^{1-a}e^{\beta(1-a)(t-s_1)}\frac{\lambda}{2\beta} + \frac{\lambda}{\beta} \\ \le& d^{1-a}e^{\beta(1-a)t}\left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right) + \frac{\lambda}{\beta} \\ &+ e^{\beta(1-a)(t-s_1)}\left(d^{1-a}\frac{\lambda}{2\beta} - \frac{\lambda}{\beta}\right) \\ \le& \left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right)d^{1-a}e^{\beta(1-a)t} + d^{1-a}\frac{\lambda}{2\beta} \end{aligned} \tag{21}$$

Repeating the above process on switching interval $[s_k, s_{k+1})$ for $\forall k \in \mathbb{N}$, one can show that $\forall t \in [s_k, s_{k+1})$

$$\omega^{1-a}(t) \le d^{(1-a)k}e^{\beta(1-a)t}\left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right) + d^{(1-a)k}\frac{\lambda}{2\beta} \tag{22}$$

Moreover, considering the fact that $s_{N_d} \le N_d\tau_m \le -N_d \ln d/(2\beta)$, we can have

$$e^{\beta(1-a)s_{N_d}} \le e^{-\frac{(1-a)N_d}{2}\ln d} = \left(d^{1-a}\right)^{-\frac{N_d}{2}} \tag{23}$$

As a result, applying (22) and (23) on $t = s_{N_d}$, one gets

$$\begin{aligned} \omega^{1-a}(s_{N_d}) &\le d^{(1-a)N_d}\left(e^{\beta(1-a)s_{N_d}}\left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right) + \frac{\lambda}{2\beta}\right) \\ &\le \left(d^{1-a}\right)^{\frac{N_d}{2}}\left(\omega_0^{1-a} - \frac{\lambda}{2\beta}\right) + d^{(1-a)N_d}\frac{\lambda}{2\beta} \\ &\le \frac{\lambda}{2\beta} \end{aligned} \tag{24}$$

The last inequality of (24) follows from condition (8). Then similar to discussions of **Case 1**, one has

$$V_{\sigma(t)}(x(t)) \equiv 0, \quad \forall t \ge \frac{1}{\beta(1-a)} + s_{N_d}$$

which implies that $\Omega(t)$ reaches zero before time

$$\frac{1}{\beta(1-a)} + N_d\tau_m \ge T(x_0, \sigma)$$

Consequently, In either case, we can conclude that $x(t) \equiv 0, \forall t \ge \frac{1}{\beta(1-a)} + N_d\tau_m$, which shows that system (4) is globally FTS under the designed switching/impulse signals $\boldsymbol{S}_{\tau_m}$ satisfying MDT condition (6) and impulsive control inputs satisfying (5). □

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Remark 2. *Different from studies in [14]–[19], the considered system (4) is essentially nonhomogeneous and contains stabilizing impulses, where each mode consists of two homogeneous vector fields with different degrees. However, condition (H_3) and (13) implies that the activated mode may be not FTS and even cause the whole system to diverge. In this case, based on impulsive control techniques, we introduce stabilizing impulses and a class of MDT impulsive-switching signals to achieve global finite-time stabilization of system (4). Theorem 1 gives a FTS criterion for system (4) involving stabilizing impulses, which provides a basis for design.*

Remark 3. *From the proof of Theorem 1, we can see that, regardless of the initial value, system (4) will certainly arrive in region $B(t) = \{x \in \mathbb{R}_n^+ | V_{\sigma(t)}^{1-a}(x(t)) \leq \lambda/(2\beta)\}$ after a finite number of stabilizing impulse jumps. Once the states enter region $B(t)$ at some point, system (4) will eventually arrive at the origin in finite time $1/(\beta(1-a))$, as shown in Fig. 3. Moreover, inequality (7) in Theorem 1 provides an estimate of settling-time $T(x_0, \sigma)$, which uncovers the relationship among impulse strength d, impulse/switching frequency τ_m and initial state x_0.*

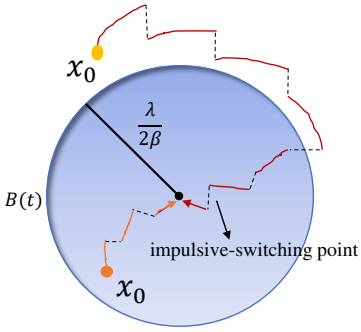

Fig. 3: Diagram of system (4) with suitable impulsive control and impulsive-switching laws.

IV. Simulation Results

In this section, a numerical simulation is provided to verify our results. Consider a class of multi-agent systems with two agents, which has been discussed in [27]. The two-agent system is described by the following SPNS with $\sigma(t) = \{1, 2\}$:

$$\begin{cases} \text{mode 1: } \dot{x}(t) = -1.3x^{\frac{2}{3}}(t) + 1.8x(t) \\ \text{mode 2: } \dot{x}(t) = -0.8x^{\frac{2}{3}}(t) + 1.6x(t) \end{cases} \tag{25}$$

where $f_1(x) = -1.3x^{\frac{2}{3}}$, $g_1(x) = 1.8x$, $f_2(x) = -0.8x^{\frac{2}{3}}$ and $g_2(x) = 1.6x$. It is easy to see that mode 1 and 2 are nonhomogeneous and satisfy Assumption 1. Moreover, all modes are not FTS for $x_0 = 6$, see Fig. 4. Let $\theta_1 = \theta_2 = 1$, we design the impulsive controllers $I_1(x) = I_2(x) = 0.4x$, i.e.,

$$x(s_k^+) = 0.4x(s_k^-) \tag{26}$$

and $\{s_k\}$ is the impulsive-switching signal to be designed. In this case, we can select $a = 2/3$, $d = 0.4$, $\lambda = 0.8$ and $\beta = 1.8$. Thus, system (25)-(26) is globally FTS for any $\{s_k\} \in S_{\tau_m}$ with $\tau_m \leq -\frac{\ln d}{2\beta} \approx 0.2545$ and the settling-time function meets $T(x_0, \sigma) \leq \frac{5}{3} + N_d\tau_m$, where N_d is defined as (8) in Theorem 1. For simulations, we take two periodic signals

$$\mathcal{S}_{0.1} : \{[0, 0.1), 1\}, \{[0.1, 0.2), 2\}, \{[0.2, 0.3), 1\}, \cdots$$
$$\mathcal{S}_{0.2} : \{[0, 0.2), 1\}, \{[0.2, 0.4), 2\}, \{[0.4, 0.6), 1\}, \cdots$$

and system state with $x_0 = 6$ is depicted in Fig. 5, which shows that the state will eventually arrive at zero after a finite number of stabilizing impulse jumps. Specifically, we select $\tau_m = 1$, which violates condition $\tau_m \leq 0.2545$. It can be seen from Fig. 6 that system (25) and (26) is unstable, which validates our results. To further verify the global stabilization capability of impulsive control and the influence of initial state value on convergence time, Fig. 7 depicts the state trajectories of system (25) under $\mathcal{S}_{0.2}$ and different initial state x_0. Meanwhile, we derive the maximum number of required impulse jumps N_d and corresponding upper bound of convergence time for system (25) under different initial conditions before converging to zero based on Theorem 1, as specified in Table I.

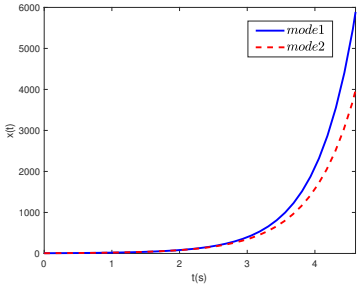

Fig. 4: State response of mode 1 and mode 2.

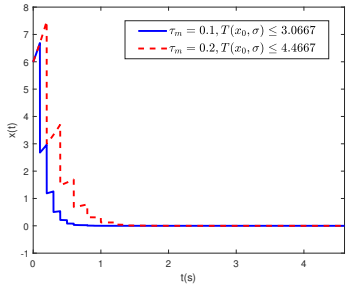

Fig. 5: State response of system with $\mathcal{S}_{0.1}$ and $\mathcal{S}_{0.2}$.

TABLE I: The comparison results of system (25) with different initial states.

x_0	N_d	τ_m	$T(x_0, \sigma)$	actual convergence time
1	9	0.2	3.467s	1.371s
5	13	0.2	4.267s	2.028s
25	17	0.2	5.067s	2.834s

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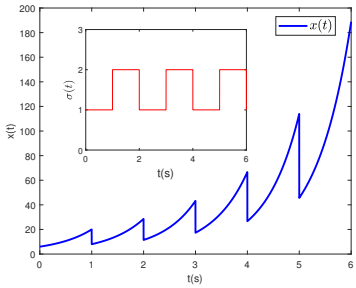

Fig. 6: State response of system with $\mathcal{S}_1$ and $x_0 = 6$.

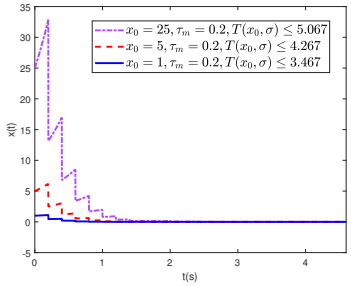

Fig. 7: State response of system with $\mathcal{S}_{0.2}$ and different x_0.

V. Conclusion

Global finite-time stabilization of switched nonhomogeneous positive nonlinear systems via impulsive control is solved in this work, where each mode consists of two homogeneous vector fields with different degrees. In this case, we guarantee the FTS property of the studied system by exerting suitable impulsive control inputs at all switching moments and designing a type of impulse/switching signals with MDT conditions. Our results show that when the system contains stabilizing impulses and impulse frequency is not lower than a constant value, the system will always converge to the origin in a finite time, no matter what the initial state. Meanwhile, we also provide an estimate of settling time, which depends on MDT constant and initial value. Moreover, we will extend the obtained results to the time-delay case in future work.

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